PROGRAM DIDIK CEMERLANG AKADEMIK

SPM

ADDITIONAL MATHEMATICS
FORM 5
MODULE 16

LINEAR PROGRAMMING
# CHAPTER 16 : LINEAR PROGRAMMING

<table>
<thead>
<tr>
<th>Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.1 Concept map</td>
<td>3</td>
</tr>
<tr>
<td>16.2 Identify and shade the region that satisfies several</td>
<td>3-8</td>
</tr>
<tr>
<td>inequalities</td>
<td></td>
</tr>
<tr>
<td>16.3 Problem interpretation and the formation of the relevant</td>
<td>9-11</td>
</tr>
<tr>
<td>equations or inequalities</td>
<td></td>
</tr>
<tr>
<td>16.4 To solve problems on linear programming using the</td>
<td>12 -</td>
</tr>
<tr>
<td>graphical method</td>
<td>15</td>
</tr>
<tr>
<td>16.5 SPM Questions</td>
<td>16 -</td>
</tr>
<tr>
<td>16.6 Assessment test</td>
<td>18-19</td>
</tr>
<tr>
<td>16.7 Answers</td>
<td>20-25</td>
</tr>
</tbody>
</table>
16.1 Concept Map

LINEAR PROGRAMMING

Region that satisfies a few linear inequalities

Problem interpretation and the formation of the relevant equations or inequalities

To solve problems on linear programming using graphical method

16.2 Identify and shade the region that satisfies several inequalities

(a) A region is said to satisfy a certain inequality if each point in the region satisfies the inequality.
(b) When the given inequality is \( ax + by + c \geq 0 \) or \( ax + by + c > 0 \) the region above the straight line \( ax + by + c = 0 \) must be shaded.

(c) A solid line (__________) line is used for an inequality which involves the sign \( \geq \) or \( \leq \) and a dashed line (______) is used for an inequality which involves the sign > or <.
When the given inequality is \( ax + by + c \leq 0 \) or \( ax + by + c < 0 \) the region \textbf{below} the straight line \( ax + by + c = 0 \) should be shaded. However, this condition is only true when the \textbf{coefficient of y on the left hand side is positive}.

\[ ax + by + c = 0 \]

\[ \begin{array}{c|c|c}
  \text{x} & 0 & 3 \\
  \text{y} & 2 & 0 \\
\end{array} \]

\textbf{Example 1}

Shade the region which satisfies the inequality \( 2x + 3y \geq 6 \).

\textit{Solution}

\textbf{Step 1}

Draw the straight line \( 2x + 3y \geq 6 \), determine the \( x \) – intercept and \( y \) – intercept.

\textbf{Step 2}

Since the inequality is \( \geq \), a full line must be drawn and the region \textbf{above} the straight line \( 2x + 3y = 6 \) must be shaded.
Example 2

Shade the region which satisfies the inequality $5x + 3y + 15 < 0$.

Solution

Step 1
Draw the straight line $5x + 3y + 15 = 0$, determine the $x$-intercept and $y$-intercept.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-5</td>
<td>0</td>
</tr>
</tbody>
</table>

Step 2
Since the inequality is $<$, a dashed line must be drawn and the region below the straight line $5x + 3y + 15 = 0$ must be shaded.
Example 3

Shade the region which satisfies the inequality $3x - 2y \leq 6$.

Solution

Step 1

Change the coefficient of $y$ on the left hand side to a positive value.

\[
3x - 2y \leq 6 \\
-3x + 2y \geq -6
\]

Step 2

Draw the line $-3x - 2y = -6$, determine the $x$– intercept and $y$– intercept.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-3</td>
<td>0</td>
</tr>
</tbody>
</table>

Step 3

Since the inequality is $\geq$, a full line is drawn and the region above this line is shaded.

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Exercise 1

1. Draw and shade the region which satisfies the following inequalities:

   (a) \( y \leq 3 - 2x \)
   (b) \( 2y > x + 6 \)

   \( 4y \geq x - 20 \)
   \( 3y < 18 - 2x \)

2. Draw and shade the region \( R \) which satisfies the following inequalities:

   (a) \( x < 5 \), \( 2y \leq x + 2 \) and \( 4x + 5y \geq 20 \)
(b) \( y > 1, \ x + y \leq 5 \) and \( 5x + 2y \geq 10 \)

3. Write down three inequalities which define the shaded region \( R \) in the diagrams below

(a)

Answer: …………………………………………………………………………..

(b)

Answer: …………………………………………………………………………..
16.3 Problem interpretation and the formation of the relevant equations or inequalities

The table below shows the mathematical expressions for the different inequalities used.

<table>
<thead>
<tr>
<th>Mathematical Expressions</th>
<th>Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>a y greater than x</td>
<td>y &gt; x</td>
</tr>
<tr>
<td>b y less than x</td>
<td>y &lt; x</td>
</tr>
<tr>
<td>c y not more than x</td>
<td>y ≤ x</td>
</tr>
<tr>
<td>d y not less than x</td>
<td>y ≥ x</td>
</tr>
<tr>
<td>e The sum of x and y is not more than k</td>
<td>x + y ≤ k</td>
</tr>
<tr>
<td>f The minimum value of y is k</td>
<td>y ≥ k</td>
</tr>
<tr>
<td>g The maximum value of y is k</td>
<td>y ≤ k</td>
</tr>
<tr>
<td>h y is at least k times the value of x</td>
<td>y ≥ kx</td>
</tr>
</tbody>
</table>

Activity 2

1. The cost of a book is 50 cents and that of a pen is RM 1.30. A student wants to buy x books and y pens with the following conditions.
   I : At least 3 pens must be bought.
   II: The total number of books and pens bought must not be more than 12.
   III: The amount of money spent is at most RM10.

Write down the three inequalities other than x ≥ 0 and y ≥ 0 that satisfy all the above conditions.

Answer : ………………………………………………………………………………………

2. A transport company delivers vegetables and flowers using lorries and vans. Each lorry carries 40 boxes of flowers and 50 boxes of vegetables. Each van carries 20 boxes of flowers and 40 boxes of vegetables. In a day, the company uses x lorries and y vans. The following conditions must be satisfied.

   I The total number of boxes of flowers must be less or equal to 800.
   II The total number of boxes of vegetables must be at least 600.
   III The number of vans must not be more than two times the number of lorries used.

   (a) Write down three inequalities other than x ≥ 0 and y ≥ 0 that satisfy all the above conditions.

Answer : ……………………………………………………………………………
(b) Using a scale of 2 cm to 5 units on both axes, draw three lines which define the region satisfying all three inequalities. Shade this region.
3. A factory employs skilled and semi-skilled workers. An amount of RM64000 is used to pay the monthly salaries of the workers. The monthly salary of a skilled worker and semi-skilled worker is RM1200 and RM800 respectively. Semi-skilled workers exceed skilled workers by at least 10 persons. The minimum number of skilled workers is \( \frac{1}{7} \) of the number of semi-skilled workers.

(a) Given that \( x \) represents the number of skilled workers and \( y \) represents the number of semi-skilled workers, write down three inequalities other than \( x \geq 0 \) and \( y \geq 0 \), that satisfy all of the above conditions.

(b) Using a scale of 2 cm to 5 workers on the \( x \)-axis and 2 cm to 10 workers on the \( y \)-axis, draw a graph to show these inequalities.
Mark and shade the region \( R \) which satisfy these conditions.

Answer

(a) ..................................................................................................................

(b) 

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16.4 To solve problems on linear programming using the graphical method

**Step 1**: Interpret the problem and form the equation or inequalities.

**Step 2**: Construct the region which satisfies the given inequalities.

**Step 3**: Determine the maximum value or minimum value \( ax + by \) from the graph by drawing the straight line \( ax + by = k \).

Example 1

A company delivers 900 parcels using \( x \) lorries and \( y \) vans. Each lorry carries 150 parcels while each van carries 60 parcels. The cost of transporting the parcels using a lorry is RM 60 while that of a van is RM 40. The total cost spent on transportation is not more than RM 480.

(a) Write down two inequalities other than \( x \geq 0 \) and \( y \geq 0 \), that satisfy all of the above conditions.

(b) Using a scale of 2 cm to 2 units on both axes, draw two lines which define the region satisfying both inequalities. Shade this region.

(c) Use your graph to find

(i) the maximum number of vans used if 5 lorries are used,
(ii) the minimum cost of transportation.

**Solution**

(a) \[
\begin{align*}
150x + 60y & \geq 900 \\
5x + 2y & \geq 30
\end{align*}
\]

(b) \[
\begin{align*}
60x + 40y & \leq 480 \\
3x + 2y & \leq 24
\end{align*}
\]

The two inequalities that satisfy the given conditions are:

\[
5x + 2y \geq 30 \quad \text{and} \quad 3x + 2y \leq 24
\]
(b) The $x$-intercept and $y$-intercept for the straight line $5x + 2y = 30$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>15</td>
<td>0</td>
</tr>
</tbody>
</table>

The $x$-intercept and $y$-intercept for the straight line $3x + 2y = 24$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \begin{align*}
5x + 2y &= 30 \\
x &= 5 \\
(5,4) \\
3x + 2y &= 24
\end{align*} \]

(c) (i) The number of lorries used is 5, so $x = 5$. Draw the line $x = 5$.
From the graph, when $x = 5$, $y = 4$ is the maximum value. Therefore the maximum number of vans used is 4.

(ii) The cost for transport, $k = 60x + 40y$, assume $k = 480$
$60x + 40y = 480$
$3x + 2y = 24$

Draw a parallel line to the straight line $3x + 2y = 24$ that passes through the shaded region and closest to the origin.
From the graph, the optimum coordinate is (6,0). Therefore the minimum cost of transportation is when 6 lorries are used.

\[
\text{The minimum cost of transportation } = RM 60 \times 6 = RM 360
\]

Exercise 3

1. A company is appointed by TV12 to make a survey and get feedback on its television programmes. The following conditions must be satisfied.

I At least 500 people surveyed must be from the rural area.
II The town people surveyed must not be less than the people from the rural area.
III The total number of people surveyed must not be more than 1500.

The company has made a survey on \(x\) rural people and \(y\) people from the town.

(a) Write down three inequalities which satisfy all the above conditions.

(b) Using a scale of 1 cm to 100 people on both axes, draw three lines which define the region satisfying all three inequalities. Shade this region.

(c) Use your graph to answer the following questions:

The company is paid RM 2 to each person surveyed. The cost to survey one rural person is RM 1.50 and RM 1.00 for a person from the town.

Find

(i) the minimum cost to make a survey by the company,
(ii) the maximum profit made by the company.

Answer:

(a) .................................................................

(c)
16.5 PAST YEAR SPM QUESTIONS

SPM 2005 Paper 2 Qn.14

An institution offers two computer courses, P and Q. The number of participants for course P is \( x \) and for course Q is \( y \).

The enrolment of the participants is based on the following constraints:

I : The total number of participants is not more than 100.

II : The number of participants for course Q is not more than 4 times the number of participants for course P.

III : The number of participants for course Q must exceed the number of participants for course P by at least 5.

(a) Write down three inequalities, other than \( x \geq 0 \) and \( y \geq 0 \), that satisfy all of the above constraints. \([3\text{marks}]\)

(b) By using a scale of 2 cm to 10 participants on both axes, construct and shade the region \( R \) that satisfies all the above constraints. \([4\text{marks}]\)

(c) By using your graph from (b), find

(i) the range of the number of participants for course Q if the number of participants for course P is 30.

(ii) the maximum total fees that can be collected if the fees per month for courses P and Q are RM50 and RM60 respectively. \([4\text{marks}]\)

Answer :

(a) ..........................................................................................................................................

(c)
16.6 Assessment

Encik Ahmad sells two types of fried noodles. A pack of plain noodles uses 120 g of prawns and 300 g of beef, while the special fried noodles uses 240 g of prawns and 200 g of beef. Encik Ahmad has 8.4 kg of prawns and 12 kg beef to make \( x \) packs of plain noodles and \( y \) packs of special noodles.

The number of packs for plain noodles cannot be more than two times the number of special noodles.

(a) Write down 3 inequalities other than \( x \geq 0 \) and \( y \geq 0 \), that satisfy all of the above constraints. [2 marks]

(b) Using a scale of 2 cm to 10 units for \( x \)-axis and 2 cm for 5 units for the \( y \)-axis, draw all the three lines for the inequalities, construct and shade the region \( R \) that satisfies all the above constraints. [3 marks]

(c) Use the graph in (b) to answer the following questions:

(i) If Encik Ahmad fries the special noodles 10 packs more than the plain noodles, state the maximum packs of plain noodles and maximum number of special noodles cooked.

(ii) What is the maximum profit obtained if a pack of plain noodles and a pack of special noodles cost RM5 and RM7 each. [5 marks]

Answer

(a) .......................................................... ..........................................................

(c)
16.7 Answers

Exercise 1

1 (a)

\[ y \leq 3 - 2x \]

(b)

\[ 2y > x + 6 \]
2 (a) $x < 5, 2y \leq x + 2, 4x + 5y \geq 20$  
(b) $y > 1, x + y \leq 5, 5x + 2y \geq 10$
3. (a)  \( x < 3, \ y \geq 0, \ y \leq 2x \) \hspace{1cm} (b)  \( y \leq 2x + 3, 3x + 5y > 15, 7x + 3y \leq 35 \)

Exercise 2

1.  \( x \geq 3, \ x + y < 12, 5x + 13y > 100 \)
2.  \( 40x + 20y \leq 800, 50x + 40y \geq 600, \ y \leq 2x \)

Activity 3

1. (a)  \( x \geq 500, \ y \geq x, \ x + y \leq 1500 \)

(c) (i) Cost \( = 1.5x + 1.0y \) . Draw the line \( 1.5x + y = 150 \) . ( \( 150 = 1.5 \times 1.0 \times 100 \) )
From the graph, the minimum cost occurs when \( x = 500, y = 500 \)
Therefore the minimum cost \( = 1.5(500) + 1.0(500) \) 
\( = RM1250 \)

(ii)  
<table>
<thead>
<tr>
<th>Area</th>
<th>Payment received</th>
<th>Cost</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rural ( ( x ) person)</td>
<td>RM 2.00</td>
<td>RM 1.50</td>
<td>RM 0.50</td>
</tr>
<tr>
<td>Town ( ( y ) person)</td>
<td>RM 2.00</td>
<td>RM 1.00</td>
<td>RM 1.00</td>
</tr>
</tbody>
</table>

Profit \( = 0.5x + 1.0y \) , draw the line \( 0.5x + y = 500 \) .
From the graph the maximum profit occurs when \( x = 500, y = 1000 \) .
Therefore the maximum profit \( = 0.5(500) + 1.0(1000) \)  
\( = RM1250 \)
\[ x = 500 \]
\[ y = x \]
\[ 0.5x + y = 500 \]
\[ x + y = 1500 \]
\[ \text{min.} \,(500,500) \]
\[ \text{max.}\,(500,1000) \]

http://mathsmozac.blogspot.com

http://sahatmozac.blogspot.com
Answer SPM 2005

(a) \( x + y \leq 100 \), \( y \leq 4x \), \( y \geq x + 5 \)

(b) \( x + y = 100 \)  
\[
\begin{array}{c|c|c}
\hline
x & 0 & 100 \\
\hline
y & 100 & 0 \\
\hline
\end{array}
\]
\[
\begin{array}{c|c|c}
\hline
x & 0 & 10 \\
\hline
y & 0 & 40 \\
\hline
\end{array}
\]
\[
\begin{array}{c|c|c}
\hline
x & 0 & 15 \\
\hline
y & 5 & 20 \\
\hline
\end{array}
\]

(c) (i) When \( x = 30, 35 \leq y \leq 70 \)

(ii) Total Fees paid, \( k = 50x + 60y \)
Let \( k = 3000, 50x + 60y = 3000 \)
\[ k_{\text{max}} \] \( x = 20, y = 80 \)
Therefore total fees paid \( = 50(20) + 60(80) \)
\( = Rm5800 \)

Assessment Test

(a) \( 120x + 240y \leq 8400 \) or \( x + 2y \leq 70 \)  
\( 300x + 200y \leq 12000 \) or \( 3x + 2y \leq 120 \)
\( x \leq 2y \) or \( y \geq \frac{1}{2}x \)

(b) Rujuk Lampiran B

(c)(i) \( y = x + 10 \)
\[ x_{\text{maximum}} = 16 \text{ and } y_{\text{maximum}} = 26 \]
Plain noodles = 16 packs
Special noodles = 26 packs

(c)(ii) \( (24, 23) \) and \( K_{\text{maximum}} = 5x + 7y \)
Maximum profit = \( 5(24) + 7(23) \)
\( = RM 281 \)
1(b)

Graph showing linear equations:

- $y = x + 10$
- $y = \frac{1}{3}x$
- $x + 2y = 70$
- $3x + 2y = 120$

Points:
- (24, 23)

Graph grid with axes labeled x and y.